Split Complex

October 26, 2023

0.1 Hyperbolic Complex Numbers and Its Properties

The set of hyperbolic numbers $\mathbb{D}$ is defined as

$$\mathbb{D} := \{x + ky : x, y \in \mathbb{R}, k^2 = 1\}.$$

Here k is called the hyperbolic unit. In some of the existing literature, hyperbolic numbers are also called duplex, double or bireal numbers.

If $\mathfrak{z}_1 = x_1 + ky_1$ and $\mathfrak{z}_2 = x_2 + ky_2$ be two hyberbolic numbers then

$$\mathfrak{z}_1 + \mathfrak{z}_2 = (x_1 + y_1) + k(x_2 + y_2)$$

and

$$\mathfrak{z}_1\mathfrak{z}_2 = (x_1x_2 + y_1y_2) + k(x_1y_2 + x_2y_1).$$

here $k^2 = 1$.

If $\mathfrak{z} = x_1 + ky_1$, then it hyperbolic conjugate, $\mathfrak{z}^\diamond := x - ky$.

$$\mathfrak{z}\mathfrak{z}^\diamond = x^2 - y^2.$$

This leads to defining hyperbolic modulus

$$|\mathfrak{z}|^2_{hyp} := x^2 - y^2.$$

Note that this is a real number and it can be negative also.

$$\mathbb{D}^+ = \{(x + ky : x^2 - y^2 \geq 0, x \geq 0\}$$

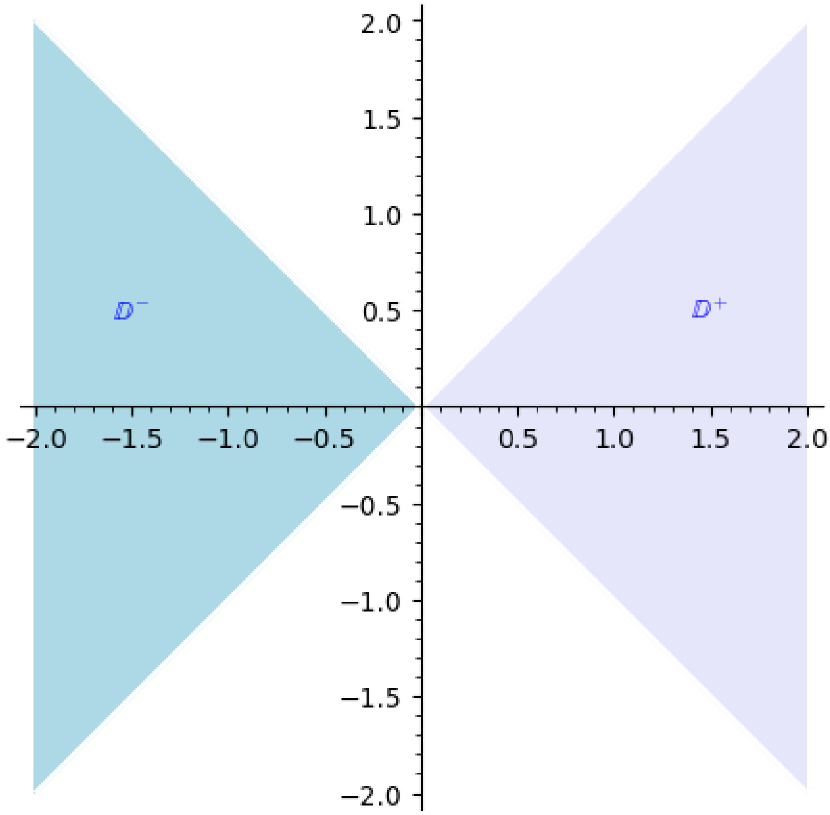
called the non negative hyperbolic numbers.

$$\mathbb{D}^- = \{(x + ky : x^2 - y^2 \geq 0, x \leq 0\}$$

called the non poitive hyperbolic numbers.

[114]:
```
var('x,y')
## Non negative hyperbolic numbers
p=region_plot([x^2-y^2>=0 and x>=0],(x,0,2),(y,-2,2),incol='lavender')
## Non positive hyperbolic numbers
q=region_plot([x^2-y^2>=0 and x<=0],(x,-2,0),(y,-2,2),incol='lightblue')
t1 = text(r'${\mathbb{D}^{+}}$',(1.5,0.5))
```

```
t2 = text(r'${\mathbb{D}^{-}}$',(-1.5,0.5))
show(p+q+t1+t2,figsize=6)
```



[1]:
```
def splitcopmplxmult(Z,W):
    return([Z[0]*W[0]+Z[1]*W[1],Z[0]*W[1]+W[0]*Z[1]])
```

[ ]:
```

```

[2]:
```
def splitpower(Z,n):
    Z0 = [1,0]
    for i in range(1,n+1):
        Z1 = splitcopmplxmult(Z0,Z)
        Z0 = copy(Z1)
    return Z1
```

[3]:
```
Z1 = [3,-2];Z2=[-1,2];
splitcopmplxmult(Z1,Z2)
```

[3]: [-7, 8]

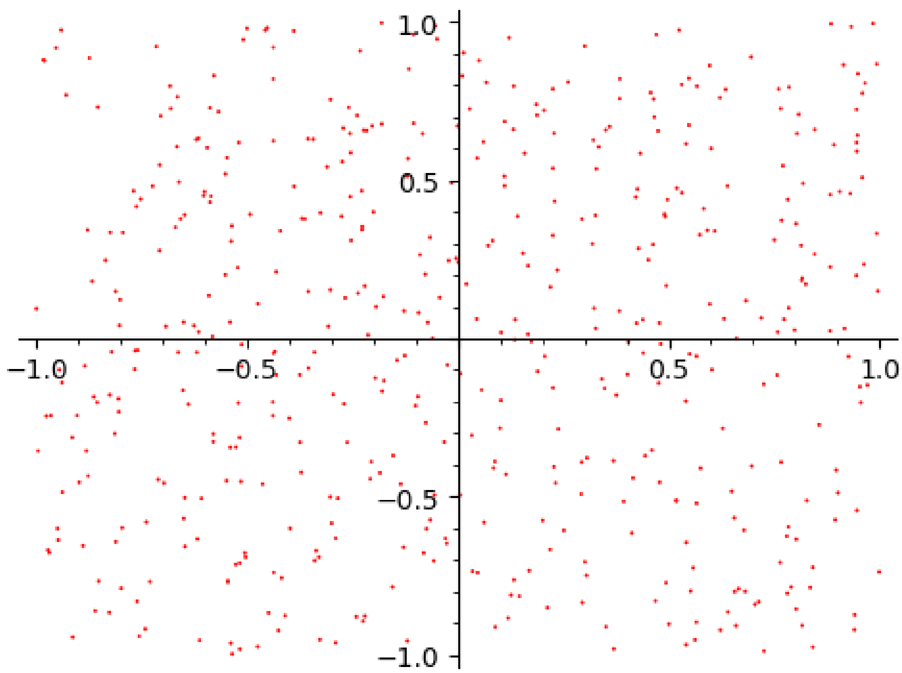
[9]:
```
splitpower([1,1],10)
```

[9]: [512, 512]

[10]:
```
Z0 = [cosh(0.05), sinh(0.05)];Z0
```

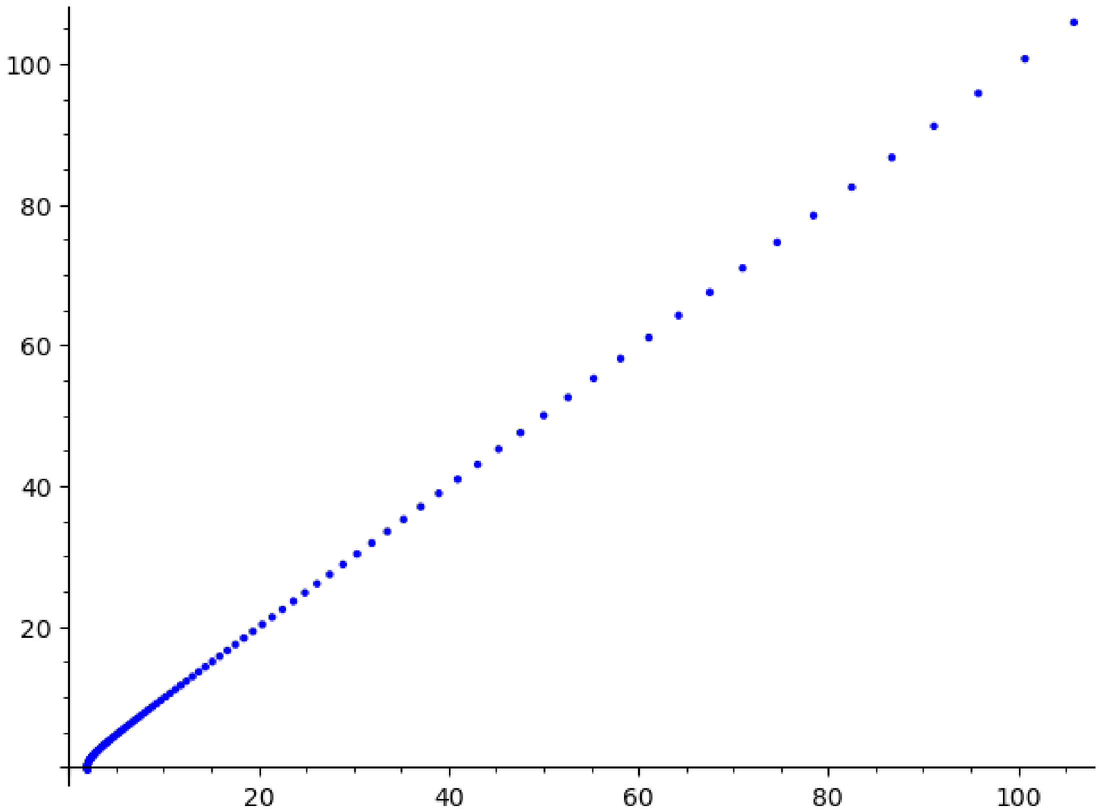
[10]: [1.00125026043837, 0.0500208359376550]

[11]:
```
pt = [[uniform(-1,1), uniform(-1,1)] for i in range(500)]
show(point2d(pt,size=2,color='red'),figsize=5)
```

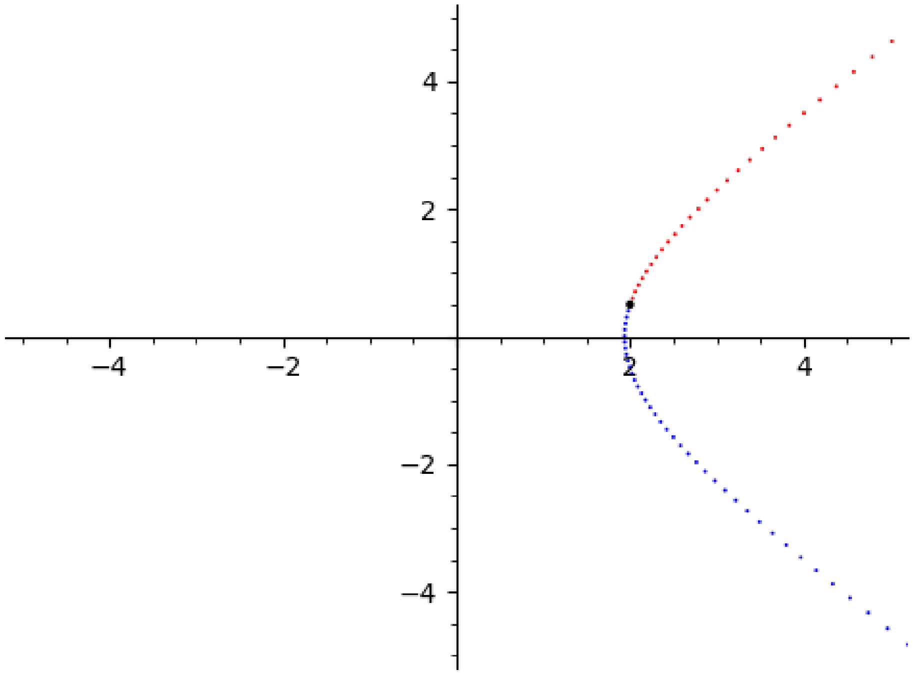


[39]:
```
Z0=[cosh(0.05),sinh(0.05)]
point2d([splitcopmplxmult([2,-0.5],splitpower(Z0,k)) for k in range(1,100)])
```

[39]:



[46]:
```
Z0=[cosh(0.05),sinh(0.05)]
Z1=[cosh(-0.05),sinh(-0.05)]
A =[2,0.5]
pA = point2d(A,color='black',size=10)
ptA=point2d([splitcopmplxmult(A,splitpower(Z0,k)) for k in␣
 ↪range(1,100)],color='red',size=2)
ptA1 = point2d([splitcopmplxmult(A,splitpower(Z1,k)) for k in␣
 ↪range(1,100)],color='blue',size=2)
show(pA+ptA+ptA1,ymax=5,ymin=-5,xmax=5,xmin=-5,figsize=5)
```

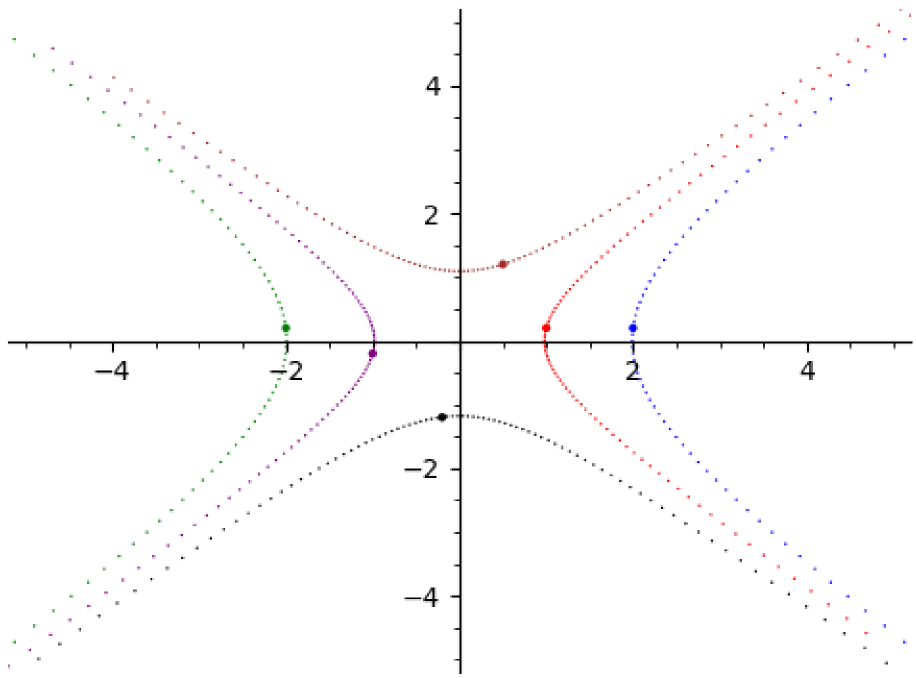


[51]:
```
Z0=[cosh(-0.05),sinh(-0.05)]
Z1=[cosh(0.05),sinh(0.05)]
A1=[1,0.2];PA1=point2d(A1,size=10,color='red')
B1 =[2,0.2];PB1=point2d(B1,size=10,color='blue')
C1 = [-2,0.2];PC1=point2d(C1,size=10,color='green')
D1 =[-1,-0.2];PD1=point2d(D1,size=10,color='purple')
E1 = [0.5,1.2];PE1=point2d(E1,size=10,color='brown')
F1 = [-0.2,-1.2];PF1=point2d(F1,size=10,color='black')
ptt = PA1+PB1+PC1+PD1+PE1+PF1
p1=point2d([splitcopmplxmult(A1,splitpower(Z0,k)) for k in␣
 ↪range(1,50)],color='red',size=1)
p2=point2d([splitcopmplxmult(A1,splitpower(Z1,k)) for k in␣
 ↪range(1,50)],color='red',size=1)
p3=point2d([splitcopmplxmult(B1,splitpower(Z0,k)) for k in␣
 ↪range(1,50)],color='blue',size=1)
p4=point2d([splitcopmplxmult(B1,splitpower(Z1,k)) for k in␣
 ↪range(1,50)],color='blue',size=1)
p5=point2d([splitcopmplxmult(C1,splitpower(Z0,k)) for k in␣
 ↪range(1,50)],color='green',size=1)
p6=point2d([splitcopmplxmult(C1,splitpower(Z1,k)) for k in␣
 ↪range(1,50)],color='green',size=1)
p7=point2d([splitcopmplxmult(D1,splitpower(Z0,k)) for k in␣
 ↪range(1,50)],color='purple',size=1)
```

```
p8=point2d([splitcopmplxmult(D1,splitpower(Z1,k)) for k in␣
 ↪range(1,50)],color='purple',size=1)

p9=point2d([splitcopmplxmult(E1,splitpower(Z0,k)) for k in␣
 ↪range(1,50)],color='brown',size=1)
p10=point2d([splitcopmplxmult(E1,splitpower(Z1,k)) for k in␣
 ↪range(1,50)],color='brown',size=1)
p11=point2d([splitcopmplxmult(F1,splitpower(Z0,k)) for k in␣
 ↪range(1,50)],color='black',size=1)
p12=point2d([splitcopmplxmult(F1,splitpower(Z1,k)) for k in␣
 ↪range(1,50)],color='black',size=1)

show(ptt+p1+p2+p3+p4+p5+p6+p7+p8+p9+p10+p11+p12,ymax=5,ymin=-5,xmax=5,xmin=-5,figsize=5)
```

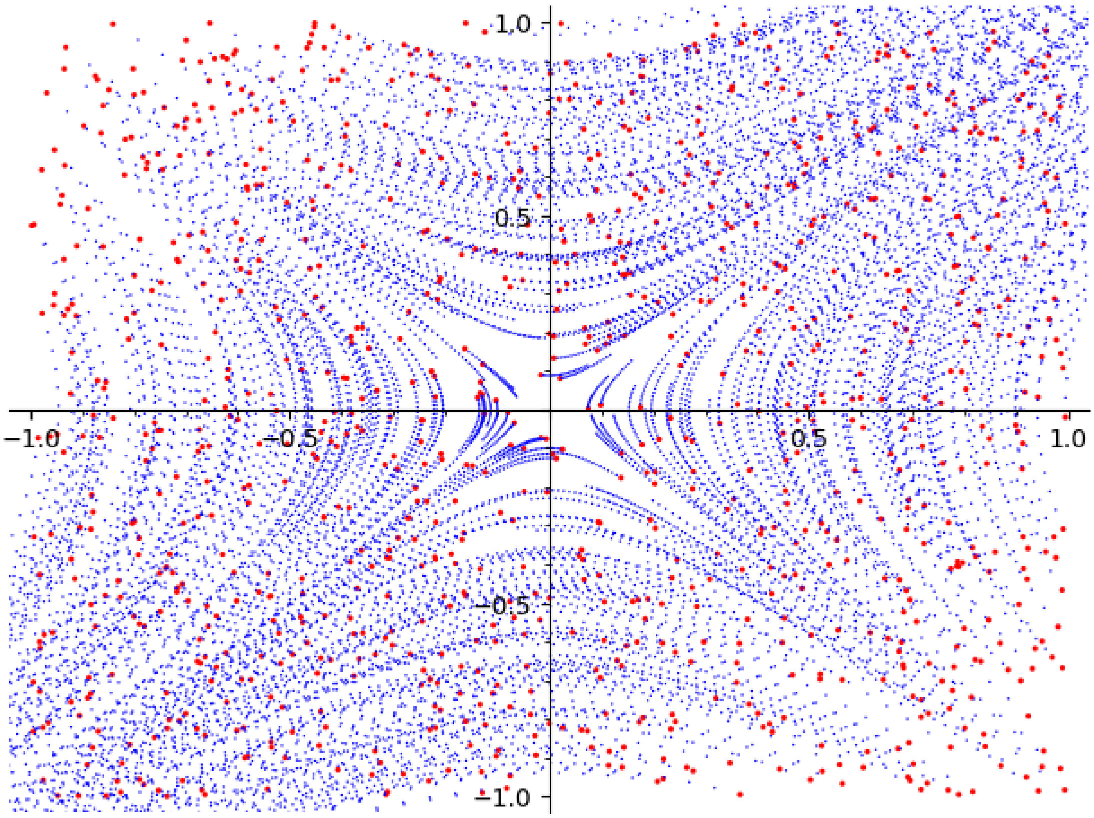


[48]:
```
N = 1000
Z0 = [cosh(0.05), sinh(0.05)];
pt = [[uniform(-1,1), uniform(-1,1)] for i in range(N)]
P0 = point2d(pt,color='red',size=5,ymax=1,ymin=-1,xmax=1,xmin=-1)
K =20
pts = []

for i in range(N):
    P = [splitcopmplxmult(splitpower(Z0,k),pt[i]) for k in range(1,K)]
    pts=pts+P
```

```
P1=point2d(pts,color='blue',size=1,ymax=1,ymin=-1,xmax=1,xmin=-1)
P0+P1
```

[48]:



[ ]: